LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2023**

UMT 6501 – COMPLEX ANALYSIS

Date: 30-10-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

Part A

Answer ALL questions: **(10 x 2 = 20)**

1. If $f(z) = \bar{z}$, then show that f is not differentiable at $z = 0$.
2. Verify Cauchy Riemann equation for the function $f(z) = Re\ z$.
3. Prove that the limit of the function is unique, if it exists.
4. Find the singular points of the function $f(z) = \frac{z^3+4}{(z^2-3)(z^2+1)}$.
5. Define harmonic function.
6. Write the Maclaurin series expansion of the function $f(z) = sinz$.
7. State maximum modulus principle.
8. Define an Essential singularity.
9. Find the residue of $\frac{z^2}{z^2+a^2}$ at $z = ai$.
10. Evaluate the zeros of $(z) = \frac{z^2+1}{1-z^2}$.

Part-B

Answer any FIVE questions: **(5 x 8 = 40)**

11. Show the function $f(z) = \sqrt{|xy|}$ is not differentiable but satisfies the Cauchy – Riemann equations.
12. State and prove the polar form of the Cauchy – Riemann equations.
13. Find the harmonic conjugate of $u(x,y) = y^3 - 3x^2y$.
14. Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Prove that $\left|\int_C \frac{z+4}{z^3-1}\, dz\right| \leq \frac{6\pi}{7}$.
15. State Liouville's theorem and deduce the Fundamental theorem of algebra.
16. Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ in a Laurent's series in (i) $1 < |z| < 2$ and (ii) $|z| > 2$.
17. State and prove Cauchy residue theorem.
18. Find the bilinear transformation which maps $z_1 = -1, z_2 = 0$ and $z_3 = 1$ onto the points $w_1 = -i, w_2 = 1$ and $w_3 = i$.

Part C

Answer any TWO questions: **(2 x 20 = 40)**

19. State and prove the necessary and sufficient condition for a function $f(z)$ to be differentiable at a point.

20. (a) If $w(t)$ is a piecewise continuous complex valued function defined on an interval $a \le t \le b$, then prove that $\left|\int_a^b w(t)dt\right| \le \int_a^b |w(t)|dt$.

(b) Verify the following functions are entire functions, using sufficient condition for differentiability.

(i) $f(z) = sinx\ coshy + i\ cosx\ sinhy$ (ii) $f(z) = (z^2 - 2)e^{-x}\, e^{-iy}$. (8+12)

21. (a) State and prove Argument principle.

(b) Using method of contour integration evaluate $\int_0^\infty \frac{dx}{1 + x^4}$

22. (a) State and prove Rouche's theorem.

(b) Determine the value of the integral $\int_C \frac{5z-2}{z(z-1)} dz$ using residue theorem where C is the circle $|z| = 2$ described counter clockwise.

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